Question 1 — Marks

a. Determine the co-ordinates of point P which divides the interval joining (3, −2) and (−5, 4) externally in the ratio 5:3. — 3

b. Solve the inequality $\frac{1}{x-2} \leq 1$ — 3

c. For the polynomial $P(x) = x^3 - 2x^2 - x + 2$

i. show that $x - 1$ is a factor. — 1

ii. Hence, or otherwise, find all the factors of $P(x)$. — 1

d. i. If $t = \tan\frac{\theta}{2}$, show that $\sin\theta = \frac{2t}{1 + t^2}$ and $\cos\theta = \frac{1 - t^2}{1 + t^2}$. — 2

ii. Using these results, show that $\frac{1 - \cos\theta}{\sin\theta} = \tan\frac{\theta}{2}$. — 1

iii. Hence find the exact value of tan 15°. — 1

Question 2 (Start a new work book)

a. For the parabola defined by the parametric equations $x = 4t$, $y = 2t^2$

i. by differentiation, show that the gradient of the tangent at the point, P, where $t = 3$, is 3. — 1

ii. find the gradient of the focal chord through P. — 1

iii. calculate the acute angle between the tangent at P and the focal chord through P. — 2

b. (i) Show that the equation $e^x = x + 2$ has a solution in the interval $1 < x < 2$. — 1

(ii) Letting $x_1 = 1.5$, use one application of Newton's Method to approximate that solution, correct to 3 decimal places. — 2

c. Six people attend a dinner party.

i. In how many different ways can they be arranged around a round table? — 1

ii. In how many different ways can they be arranged if a particular couple must sit together? — 1

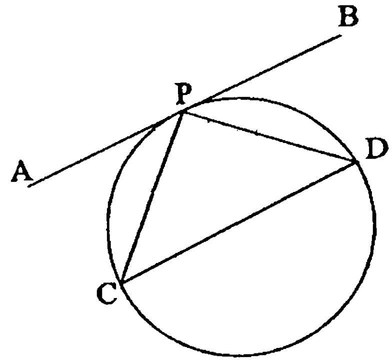
iii. What is the probability that, if the people are seated at random, the couple are sitting apart from each other? — 1

Marks

Question 2 (continued)

d. PC and PD are equal chords of a circle. A tangent, AB, is drawn at P. 2

Prove that AB is parallel to CD



Question 3

a. Jane, a netball goal shooter, has a 70% probability of scoring a goal at any attempt. In her next 10 attempts at scoring, what is the probability that she scores at least 8 times? Give your answer as a decimal to 2 significant figures. 3

b. Find the maximum value of $3\cos x - 2\sin x$ 2

c. Use the Principle of Mathematical Induction to prove that $2^{3n} - 3^n$ is divisible by 5 for all positive integers n. 4

d. The arc of the curve $y = \cos 2x$ between $x = 0$ and $x = \frac{\pi}{6}$ is rotated through 360° about the x-axis. 3

Find the exact volume of the solid formed.

Question 4

a. If $\binom{n}{r} = \binom{n}{r+1}$, where n and r are positive integers, show that n is odd. 3

b. i. Express $x^2 + 6x + 13$ in the form $(x + a)^2 + b^2$ 1

ii. Hence, using the substitution $u = x + 3$, find $\int \frac{dx}{x^2 + 6x + 13}$ 2

Marks

Question 4 (continued)

c. Show that $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$ **3**

d. Find the coefficient of x^4 in the expansion of $\left(2x - \frac{3}{x^2}\right)^{10}$. **3**

Question 5

a. A particle's motion is defined by the equation $v^2 = 12 + 4x - x^2$, where x is its displacement from the origin in metres and v its velocity in ms^{-1}. Initially, the particle is 6 metres to the right of the origin.

i. Show that the particle is moving in Simple Harmonic Motion **1**

ii. Find the centre, the period and the amplitude of the motion **3**

iii. The displacement of the particle at any time t is given by the equation $x = a\sin(nt + \theta) + b$.

Find the values of θ and b, given $0 \leq \theta \leq 2\pi$ **2**

b. Newton's Law of Cooling states that the rate of change in the temperature, $T°$, of a body is proportional to the difference between the temperature of the body and the surrounding temperature, $P°$.

i. If A and k are constants, show that the equation $T = P + Ae^{kt}$ satisfies Newton's Law of Cooling. **2**

ii. A cup of tea with a temperature of 100°C is too hot to drink. Two minutes later, the temperature has dropped to 93°C. If the surrounding temperature is 23°C, calculate A and k. **2**

iii. The tea will be drinkable when the temperature has dropped to 80°C. How long, to the nearest minute, will this take? **2**

Question 6

Marks

a. A particle is projected horizontally with velocity, V ms^{-1}, from a point h metres above the ground. Take g ms^{-2} as the acceleration due to gravity.

i. Taking the origin at the point on ground immediately below the projection point, find expressions for x and y, the horizontal and vertical displacements respectively of the particle at time t seconds. **2**

ii. Hence show that the equation of the path of the particle is given by the equation $y = \dfrac{2hV^2 - gx^2}{2V^2}$. **2**

iii. Find how far the particle travels horizontally from its point of projection before it hits the ground. **2**

b. A particle moves in a straight line so that its velocity after t seconds is v ms^{-1} and its displacement is x.

i. Given that $\dfrac{d^2x}{dt^2} = 10x - 2x^3$ and that $v = 0$ when $x = -1$, find v in terms of x. **3**

ii. Explain why the motion cannot exist between $x = -1$ and $x = 1$. **2**

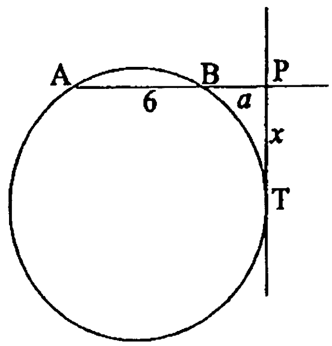
iii. Describe briefly what would have happened if the motion had commenced at $x = 0$ with $v = 0$. **1**

Question 7 (Start a new work book)

a. In the circle, the chord AB is 6 metres long. The chord is produced to the point P and BP is a metres.

A tangent to the circle cuts the chord at P. PT is x metres.

Show that $x = \sqrt{a(a + 6)}$. **2**



Question 7 (continued) **Marks**

b. A water tank is generated by rotating the curve

$$y = \frac{x^4}{16}$$

around the y - axis.

i. Show that the volume of water, V as a function of its depth h, is given by: **2**

$$V = \frac{8}{3}\pi.h^{\frac{3}{2}}$$

ii Water drains from the tank through a small hole at the bottom. **4**

The rate of change of the volume of water in the tank is proportional to the square root of the water's depth.

Use this fact to show that the water level in the tank falls at a <u>constant rate.</u>

c. i Sketch $y =$ cosec x in the domain $-\pi < x < \pi$. **1**

ii Explain why the inverse of $y =$ cosec x is not a function over this domain. **1**

iii $y = \text{cosec}^{-1} x$ is the inverse of $y =$ cosec x where the domain is restricted to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ but excluding $x = 0$. **2**

Sketch $y = \text{cosec}^{-1} x$ and state its domain and range.

End of paper

YEAR 12 TRIAL HSC Manly High EXTENSION

2001

Q1

a) $P\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$

$m:n = -5:3$

$P\left(\frac{34}{-2}, \frac{-26}{-2}\right)$

$= P(-17, 13)$

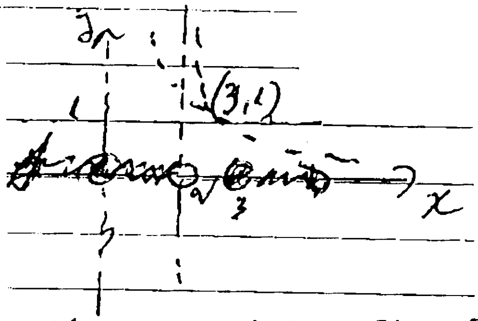
"3" correctly applied rule for ext ÷ both 'x' and 'y' correct

"2" correct formula with one numeri error

"1" two numerical e

"1" recognised ratio $-5:3$ or $5:-$

(b) $x \neq 2$



$x<2$ or $x \geq 3$

or

$x-2 \leq (x-2)^2$ $\quad x \neq 2$

$x-2 \leq x^2-4x+4$

$0 \leq x^2-5x+6$

$0 \leq (x-3)(x-2)$

$x<2$ or $x \geq 3$

"3" used graphical or algebraic means to sol correctly

"2" all correct exce $x \neq 2$

"2" solved between $2 < x \leq 3$

"1" either $x<2$ $x \geq 3$

c) i) $P(1) = 1-2-1+2 = 0$

or polynomial division

"1"

ii)

$$\begin{array}{r} x^2-x-2 \\ x-1 \enclose{longdiv}{x^3-2x^2-x+2} \\ \hline 0 \end{array}$$

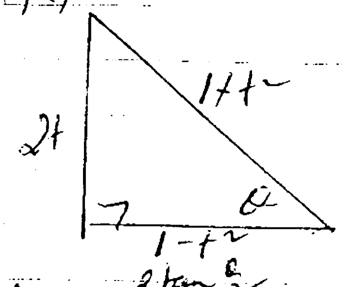
"1" use ÷ or factor theorem

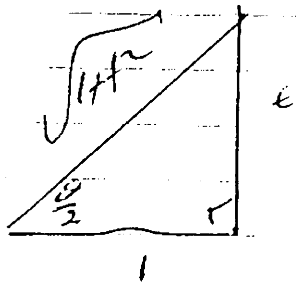
$P(x) = (x-1)(x^2-x-2)$

$= (x-1)(x-2)(x+1)$

"2"

d) i)





$\tan\theta = \dfrac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$

$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\cos\theta = \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$

$= \dfrac{2t}{1-t^2}$ thus $\sin\theta = \dfrac{2t}{1+t^2}$

$\sin\theta =$

$\cos\theta =$ $\qquad \cos\theta = \dfrac{1-t^2}{1+t^2}$

"2" establish an appropriate $\triangle$ for either 'θ' '$\frac{\theta}{2}$' and then correctly read / calculate $\sin\theta$ $\cos\theta$.

"1" for correct $\triangle$

"1" for not showing $\tan\theta = \dfrac{2t}{1-t^2}$

ii) $\dfrac{1-\cos\theta}{\sin\theta} = \dfrac{1-\frac{1-t^2}{1+t^2}}{\frac{2t}{1+t^2}} \times \dfrac{1+t^2}{1+t^2}$

$= \dfrac{1+t^2-1+t^2}{2t}$

$= \dfrac{2t^2}{2t}$ "1"

$= t$

Using these results

iii) $\tan 15^\circ = \dfrac{1-\cos 30^\circ}{\sin 30^\circ}$

$= \dfrac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} \times \dfrac{2}{2}$

$= 2-\sqrt{3}$ "1"

N.B Hence means only this method permissable

Question 2

a) $x = 4t \qquad y = 2t^2$

(A)

i) $\frac{dx}{dt} = 4 \qquad \frac{dy}{dt} = 4t$ OR $t = \frac{x}{4}$

1

$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

$= \frac{4t}{1} \times \frac{1}{4}$

$= t$

when $t = 3$ $\frac{dy}{dx} = 3$.

$y = 2\left(\frac{x}{4}\right)^2$

$y = 2\left(\frac{x^2}{16}\right)$

$y = \frac{x^2}{8}$

$\frac{dy}{dx} = \frac{x}{4}$

$x = 4(3)$

$= 12$

$\therefore \frac{dy}{dx} = \frac{12}{4}$

$= 3.$

$\frac{1}{2}$ $\frac{dy}{dx} = t$

1 $\frac{dy}{dx} = 3$

OR

$\frac{1}{2}$ $\frac{dy}{dx} = \frac{x}{4}$

1 $\frac{dy}{dx} = 3$

ii) $x^2 = 8y$

1

$x^2 = 4ay \rightarrow 4a = 8$

$a = 2$

$\therefore$ focus $(0,2) \qquad P(12,18)$

$m = \frac{18-2}{12-0}$

$= \frac{16}{12}$

$m = \frac{4}{3}$

$\frac{1}{2}$ focus $(0,2)$ or point $(12,18)$

1 $m = \frac{4}{3}$.

iii) $\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| \qquad m_1 = 3 \quad m_2 = \frac{4}{3}$

2

$= \frac{3 - \frac{4}{3}}{1 + 3\left(\frac{4}{3}\right)}$

$= \frac{1}{3}$

$\theta = \tan^{-1}\frac{1}{3}$

$\theta = 18^\circ 26'$

$\frac{1}{2}$ formula

1 correct substitution with values

2 answer.

$1\frac{1}{2}$ $\frac{1}{3}$ or what

b) i) let $f(x) = e^x - x - 2$. $x+2-e^x$

③ 1 $f(1) = e^1 - 1 - 2$

$= -0.28$ -0.217

$f(2) = e^2 - 2 - 2$ — $\frac{1}{2}$ for 1 value ca

$= 3.389\ldots$ 3.3891 2nd same diff sign

as $f(1) < 0$ & $f(2) > 0$ there is — corr. conc.

at least 1 solution in $1 < x < 2$.

ii) $f(x) = e^x - x - 2$ $f'(x) = e^x - 1$ ($\frac{1}{2}$ correct form.)

2 $f(1.5) = e^{1.5} - 1.5 - 2$ $f'(1.5) = e^{1.5} - 1$ — $\frac{1}{2}$ $f'(x)$ or sub $f(x)$

$= e^{1.5} - 3.5$ $=$ — 1 sub corr. $f(x)$, $f'(x)$

$1\frac{1}{2}$ corr. ans

$$x_2 = x_1 - \frac{f(x)}{f'(x)}$$ — 2 corr. soln. 3dp

$$= 1.5 - \frac{e^{1.5} - 3.5}{e^{1.5} - 1}$$

$= 1.21804\ldots$ $\frac{1}{2}$

$= 1.218.$ 2.

c) 1 i) $5! = 120$ — 1

③

1 ii) sit couple (sit rest) AB BA

$4! \times 2 = 48$ — 1

1 iii) ways apart $= 120 - 48$

$= 72$ — $\frac{1}{2}$ 72 sitting apart

$\therefore$ Prob (sit apart) $= \frac{72}{120}$ — $\frac{1}{2}$ prob OR

$= \frac{3}{5}$

d) $A\hat{P}C = P\hat{D}C$ OR $B\hat{P}D = P\hat{C}D$ (angle btn tang. & chord = $\angle$ alt seg) — $\frac{1}{2}$ base angles

$P\hat{C}D = P\hat{D}C$ (base $\angle$ isos Δ) — 1 also tang/chord st. corr.

$A\hat{P}C = P\hat{C}D$ OR $P\hat{D}C = B\hat{P}D$ (alt $\angle$s : AB ∥ CD) — 2 conc.

Question 3

a) p = prob goal 0.7
(i) q = " missing 0.3
$n = 10$

P(at least 8)
$= P(8) + P(9) + P(10)$
$= \binom{10}{8}0.3^2(0.7)^8 + \binom{10}{9}0.3(0.7)^9 + \binom{10}{10}0.7^{10}$
$\quad 0.23347 \qquad 0.121\ldots \qquad 0.0282.$
$= 0.38278\ldots$
$= 0.38$ (2 sig figs)

1 $n=10, p=0.7, q=0.3$
1 $\binom{10}{2}0.3^2 \cdot 7^2 = 0.23$ 2 sig figs
1 others 2 sig figs
(½ recog
½ doing

b) $3\cos x - 2\sin x = R\cos(x+\alpha)$
(ii) where $R = \sqrt{3^2+2^2}$
$R = \sqrt{13} \qquad (3.6055)$

1 $R\cos(x+\alpha)$
1 $R = \sqrt{13}$ ½ max is
1 $\frac{dy}{dx} = 0 \quad \tan^{-1}(-\frac{2}{3})$
$y = 3.6$ 1 check max.

c) $2^{3n} - 3^n$ divisible by 5.
(iii) for $n=1$ $2^3 - 3^1 = 5$
$\therefore$ true for $n=1$.
Assume true for $n=k$
$2^{3k} - 3^k = 5P$ for integer P
Show true for $n=k+1$
$2^{3(k+1)} - 3^{k+1} = 5Q$ for integer Q
$LHS = 2^{3k}.8 - 3^k.3$
$= 8(5P + 3^k) - 3.3^k$
$= 40P + 8.3^k - 3.3^k$
$= 40P + 5.3^k$
$= 5(8P + 3^k)$
$= 5Q$ where $Q = 8P + 3^k$
$\therefore$ if true for $n=k$, then true for $n=k+1$
True for $n=1$, therefore true for $n=2$. If true for $n=2$, then true for $n=3$ and so on for all positive integers n

1 check $n=1$
1 ass. + $k+1$
1 $5Q$
1

3b - alternative.

$y = 3\cos x - 2\sin x$

$\frac{dy}{dx} = -3\sin x - 2\cos x$

$\frac{d^2y}{dx^2} = -3\cos x + 2\sin x$

for st. value $\frac{dy}{dx} = 0$

$-3\sin x - 2\cos x = 0$ $\frac{1}{2}$

$-3\sin x = 2\cos x$

$\tan x = -\frac{2}{3}$

$x = \tan^{-1}(-\frac{2}{3})$ 1

$y = 3\cos(\tan^{-1}\frac{2}{3}) - 2\sin(\tan^{-1}(-\frac{2}{3}))$

$y =$ use radians or degrees

$= 3.6055..$

check using 2nd deriv. or +/- 1st.

$x = \tan^{-1}(-\frac{2}{3})$

$\frac{d^2y}{dx^2} < 0$ $\therefore$ maximum. 2

$\frac{1}{2}$ $\frac{dy}{dx} = 0$

1 $\tan^{-1}(-\frac{2}{3})$

$1\frac{1}{2}$ y-value $y = 3.6055...$ no deduction for point.

2 : checking as max.

d) $y = \cos 2x \quad 0 \to \frac{\pi}{6}$

3

$$y^2 = \cos^2 2x$$

$$\cos 2x = 2\cos^2 x - 1$$
$$\cos^2 x = \tfrac{1}{2}\cos 2x + \tfrac{1}{2}$$
$$\cos^2 2x = \tfrac{1}{2}\cos 4x + \tfrac{1}{2}$$

$$V = \pi \int y^2\,dx$$

$$= \pi \int_0^{\frac{\pi}{6}} \cos^2 2x \, dx \qquad \tfrac{1}{2}$$

$$= \pi \int_0^{\frac{\pi}{6}} \left(\tfrac{1}{2}\cos 4x + \tfrac{1}{2}\right) dx \qquad 1$$

$$= \frac{\pi}{2} \int_0^{\frac{\pi}{6}} (\cos 4x + 1)\, dx$$

$$= \frac{\pi}{2} \left[\frac{\sin 4x}{4} + x \right]_0^{\frac{\pi}{6}} \qquad 1$$

$$= \frac{\pi}{2} \left\{ \left(\frac{\sin 4(\frac{\pi}{6})}{4} + \frac{\pi}{6} \right) - (\sin 0 \cdot 0) \right\} \leftarrow \tfrac{1}{2} \text{ sub.}$$

$$= \frac{\pi}{2} \left(\frac{1}{4} \times \sin \frac{2\pi}{3} + \frac{\pi}{6} \right)$$

$$= \frac{\pi}{2} \left(\frac{1}{4} \times \frac{\sqrt{3}}{2} + \frac{\pi}{6} \right)$$

$$= \frac{\pi}{2} \left(\frac{\sqrt{3}}{8} + \frac{\pi}{6} \right)$$

$$= \frac{\pi}{4} \left(\frac{\sqrt{3}}{4} + \frac{\pi}{3} \right)$$

$$\text{Volume} = \frac{\pi}{4} \left(\frac{3\sqrt{3} + 4\pi}{12} \right) \text{ units}^3 \qquad \tfrac{1}{2}$$

3 Unit Trial 2001

Question 4

a.) $\binom{n}{r} = \binom{n}{r+1}$

$\frac{n!}{r!(n-r)!} \left(\frac{1}{2}\right) = \frac{n!}{(r+1)!(n-r-1)!} \left(\frac{1}{2}\right)$ 1

(3) $\frac{(n-r-1)!}{(n-r)!} = \frac{r!}{(r+1)!}$ $(n)!$ taken from both sides.

$\frac{(n-r-1)!}{(n-r)(n-r-1)!} \left(\frac{1}{2}\right) = \frac{r!}{(r+1)r!} \left(\frac{1}{2}\right)$ 1

$\frac{1}{n-r} = \frac{1}{r+1}$ $\cancel{\left(\frac{1}{2}\right)}$

$n-r = r+1$

$n = 2r+1$ $\left(\frac{1}{2}\right)$

since r is a positive integer, n is odd $\frac{1}{2}$

b.) i.) $x^2+6x+13 = (x+3)^2 + 4 + 9$ (1)

$= (x+3)^2 + 2^2$ (1)

ii.) $\int \frac{dx}{x^2+6x+13}$ Let $u = x+3$

$\frac{du}{dx} = 1$

$dx = du$

$= \int \frac{dx}{(x+3)^2+2^2}$

$= \int \frac{du}{u^2+2^2}$ (1)

(2) $= \frac{1}{2}\tan^{-1}\frac{u}{2} + c$

$= \frac{1}{2}\tan^{-1}\frac{x+3}{2} + c.$ (1)

Question 4 (con't)

c.) $\cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}$ $\qquad \frac{1}{2}$ $\qquad \cos\alpha = \frac{4}{5} \quad \cos\beta = \frac{3}{5}$

$\therefore \sin\alpha = \frac{3}{5} \qquad \sin\beta = \frac{4}{5} \qquad \frac{1}{2}$

(3) $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \frac{4}{5} \times \frac{3}{5} - \frac{3}{5} \times \frac{4}{5} \qquad \frac{1}{2}$

$= \frac{12}{25} - \frac{12}{25}$

$= 0 \qquad \frac{1}{2}$

$\cos(\alpha+\beta) = 0$

$\alpha+\beta = \frac{\pi}{2}$

$\therefore \cos^{-1}\left(\frac{4}{5}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2}.$

d.) $\left(2x - \frac{3}{x^2}\right)^{10}$

(3) $T_{k+1} = {}^{n}C_{k}\, a^{n-k}\, b^{k}$ $\qquad \frac{1}{2}$

$= {}^{10}C_{k}\, (2x)^{10-k} \left(-\frac{3}{x^2}\right)^{k}$

$= {}^{10}C_{k}\, 2^{10-k}\, x^{10-k}\, (-3)^{k}\, x^{-2k} \qquad \frac{1}{2}$

$x^{10-k} \times x^{-2k} = x^4$

$x^{10-3k} = x^4 \qquad \frac{1}{2}$

$10 - 3k = 4$

$6 = 3k \qquad \frac{1}{2}$

$k = 2.$

$T_{2+1} = {}^{10}C_{2}\, 2^{8}\, (-3)^{2}\, x^4$

$= 45 \times 256 \times 9$

$= 103680 \qquad (1)$

3 Unit Trial 2001
Question 5A

i.) $v^2 = 12 + 4x - x^2$

i.) $a = \left[\dfrac{d\frac{1}{2}v^2}{dx}\right]$

$\frac{1}{2}v^2 = 6 + 2x - \frac{x^2}{2}$ $\left(\frac{1}{2}\right)$

$\ddot{x} = 2 - x$

$= -1(x-2)$

$\therefore$ SHM

in form $\ddot{x} = -n^2X$

where $n = 1$ & $X = (x-2)$

ii.) centre at $\ddot{x} = 0$ $\left(\frac{1}{2}\right)$

$0 = 2 - x$

$\therefore x = 2.$ $\left(\frac{1}{2}\right)$

period $\frac{2\pi}{n}$ $n = 1$ $\left(\frac{1}{2}\right)$ $\therefore$ 2π $\left(\frac{1}{2}\right)$

amplitude at $v = 0$ $\left(\frac{1}{2}\right)$

$0 = 12 + 4x - x^2$

$= (6-x)(2+x)$

$x = 6$ or -2

$\therefore$ amplitude is $\frac{6-2}{}$ $\left(\frac{1}{2}\right)$ 4

iii.) $x = a\sin(nt + \theta) + b$

$a = 4$ $n = 1$
centre or $b = 2$ $\left(\frac{1}{2}\right)$

$x = 4\sin(t + \theta) + 2.$

when $t = 0$ $x = 6$ so $6 = 4\sin\theta + 2$ $\left(\frac{1}{2}\right)$

$4 = 4\sin\theta$

$\sin\theta = 1$ $\left(\frac{1}{2}\right)$

$\theta = \frac{\pi}{2}.$

$\therefore$ $x = 4\sin\left(t + \frac{\pi}{2}\right) + 2.$

Question 5b

b) $\frac{dT}{dt} = k(T-P)$ Newtons law

i) $T = P + Ae^{kt}$ where $Ae^{kt} = T-P$

$\frac{dT}{dt} = kAe^{kt}$ 1 — 1 kAe^{kt}

$= k(T-P)$ as required 2 — 2 $k(T-P)$

ii) $t=0$, $T=100$ $P=23$

$100 = 23 + Ae^{0}$

$77 = A$ — 1 A value

$\therefore T = 23 + 77e^{kt}$

when $t=2$, $T=93$

$93 = 23 + 77e^{2k}$

$70 = 77e^{2k}$

$e^{2k} = \frac{70}{77}\ \frac{10}{11}$

$2k = \log_e \frac{10}{11}$

$k = \frac{1}{2}\log_e \frac{10}{11}$ — 1 k value

$= -0.04765\ldots\ldots$

iii) $80 = 23 + 77e^{kt}$

$57 = 77e^{kt}$

$kt = \log_e \frac{57}{77}$

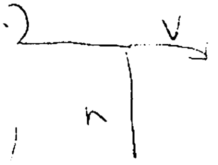
$t = \frac{\log_e \frac{57}{77}}{k}$ — 1 getting to correct log

$= 6.311\ldots\ldots$ min — 1 6.311..min

Time is 6 minutes.

3 Unit Trial 2001

Question 6.

a) i) 

2

$t = 0$ $\quad \dot{x} = V\cos\theta \quad \dot{y} = V\sin\theta$

$\dot{x} = V \quad \dot{y} = 0$

$x = 0 \quad y = h$

$\ddot{x} = 0$

$\dot{x} = c_1$ $\quad \frac{1}{2}$

$t=0 \quad \dot{x} = V \quad \therefore c = V$

$\dot{x} = V$

$x = Vt + c_2$

$t=0 \quad x=0 \quad \therefore \underline{x = Vt}$ $\quad \frac{1}{2}$

$\ddot{y} = -g$

$\dot{y} = -gt + k_1$ $\quad \frac{1}{2}$

$t=0 \quad \dot{y}=0 \quad k = 0$

$\dot{y} = -gt$

$y = \frac{-gt^2}{2} + k_2$

$t=0 \quad y=h \quad \therefore k = h$

$y = \frac{-gt^2}{2} + h$ $\quad \frac{1}{2}$

ii) $t = \frac{x}{V}$ $\quad \frac{1}{2}$

2

$y = \frac{-g}{2}\left(\frac{x}{V}\right)^2 + h$ $\quad \frac{1}{2}$

$= \frac{-gx^2}{2V^2} + h$ $\quad \frac{1}{2}$

$= \frac{-gx^2 + 2V^2h}{2V^2} \quad = \frac{2V^2h - gx^2}{2V^2}$ $\quad \frac{1}{2}$

iii): find x at $y = 0$ time

$0 = \frac{2V^2h - gx^2}{2V^2}$ $\quad \frac{1}{2}$

2

$= 2V^2h - gx^2$

$gx^2 = 2V^2h$ $\quad \frac{1}{2}$

$x^2 = \frac{2V^2h}{g}$

$x = \pm\sqrt{\frac{2V^2h}{g}}$

$= \pm V\sqrt{\frac{2h}{g}}$ $\quad \frac{1}{2}$

moving in $+$ direction $\frac{1}{2}$

$\therefore x = +V\sqrt{\frac{2h}{g}}$

3 Unit Trial 2001

Question 6 (con't)

b.) i.) $\dfrac{d^2x}{dt^2} = 10x - 2x^3$ $\quad v = 0$ when $x = -1$

$\dfrac{d}{dx}\left(\tfrac{1}{2}v^2\right) = 10x - 2x^3$

$\tfrac{1}{2}v^2 = \int 10x - 2x^2 \, dx$

$= 5x^2 - \dfrac{2x^4}{4} + c$ 1

(3)

$v^2 = 10x^2 - x^4 + c$

$0 = 10 - 1 + c$ $\tfrac{1}{2}$

$c = -9$

$v^2 = 10x^2 - x^4 - 9$ $\tfrac{1}{2}$

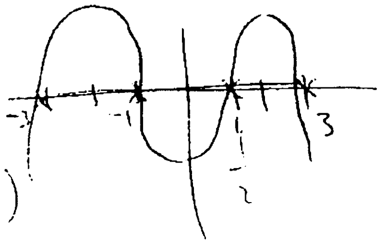
$v = \pm\sqrt{10x^2 - x^4 - 9}$ $\tfrac{1}{2}$

ii.) $v^2 = -(x^4 - 10x^2 + 9)$

$= -(x^2-9)(x^2-1)$ $\tfrac{1}{2}$

(2)

$= -(x-3)(x+3)(x-1)(x+1)$ $\tfrac{1}{2}$



$v = 0$ at $\pm 3 \pm 1$

between -1 & 1 $\quad v^2 < 0$ or $v < 0$ $\tfrac{1}{2}$

so motion can't exist

iii.) If $x = 0$ then acceleration also 0

$\dfrac{d^2x}{dt^2} = 10(0) - 2(0)^3$

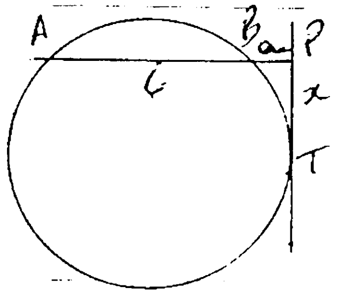
(1)

$= 0$

if $v = 0$ no movement could have occurred ∴ particle at rest (stationary) (1)

Q7

a)



$PT^2 = PB \times PA$ (pwr of circle) "1"

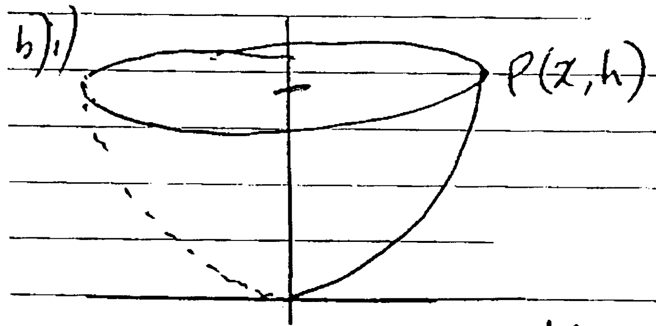
$x^2 = a \times (a+6)$

$x = \sqrt{a(a+6)}$ "1"

"2" needed to quote property in terms of PT, PA and P for them to solve for x

"1" not quoting prop

b) i)



$x^4 = 16y$

$x^2 = 4y^{\frac{1}{2}}$

"2" for correct calc of x^2 and correct $\int$

"1" for incorrect x^2 but then integration OK

$V_x = \pi \int_0^h x^2 \, dy$

$= \pi \int_0^h 4y^{\frac{1}{2}} \, dy$

$= 4\pi \left[\frac{2}{3} y^{3/2} \right]_0^h$

$= 4\pi \left(\frac{2}{3} h^{3/2} - 0 \right)$

$= \frac{8\pi}{3} h^{3/2} \; u^3$

ii) $\frac{dV}{dt} = -k\sqrt{h}$ (1) no penalty $= k\sqrt{h}$

"4" for $\frac{dV}{dt}$ chain rule $\frac{dV}{dh}$ and cancelling

$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ (1)

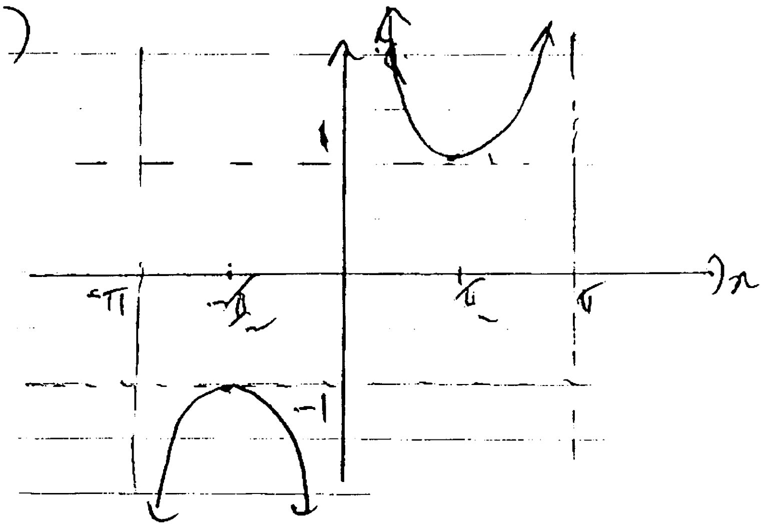
$= \frac{dV}{dt} \div \frac{dV}{dh}$ NB $\frac{dV}{dh} = 4\pi h^{\frac{1}{2}}$ (1)

$\frac{1}{2}$ for $\frac{dV}{dt} = \sqrt{h}$

$= \frac{-k\sqrt{h}}{4\pi\sqrt{h}}$ &

$= -k/4\pi$ constant (1)

c) i)



Freehand to incl. asymptotes as $x = 0$ $x = \pm \pi$ and turning pts $(\frac{\pi}{2}, 1)$ $(-\frac{\pi}{2}, -$

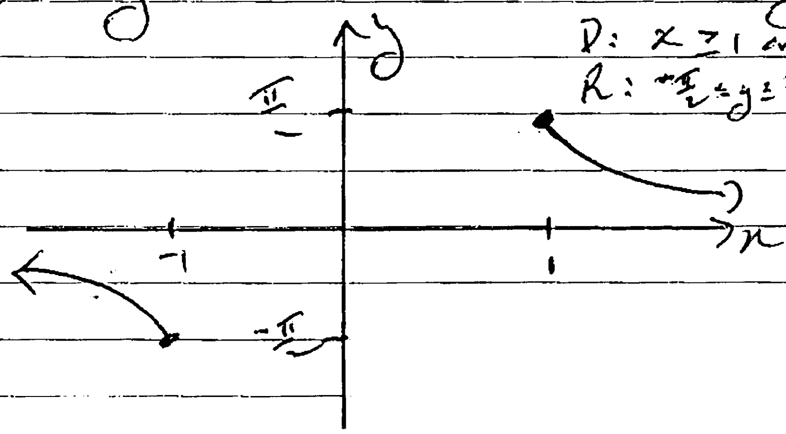
"1"

ii) fails horizontal line test or statement to that effect "1"

iii) $y = \operatorname{cosec}^{-1} x \Rightarrow x = \operatorname{cosec} y$

D: $x \geq 1$ or $x \leq -1$

R: $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, $y \neq 0$



"1" for sketch

"$\frac{1}{2}$" each for correct Domain and Ra

NB If graph wr but Domain and Range read correct of mark